**GREENWOOD COLLEGE**

**Mathematics Methods Units 3 & 4**

**Test 4 Discrete Probability Distributions 2019**

Name Marking Key Mark /18

**All electronic devices must be switched off and in bags.**

**Access to Formulae Sheet allowed. No notes.**

**No calculators allowed in this section. Time limit 20 minutes.**

1. [ 1,1,3 = 5 marks]

Let , if there is rain tomorrow and if there is no rain tomorrow, the probability of rain tomorrow being 0.1.

a) What type of probability distribution is this?

Bernoulli distribution ✓

b) Construct the probability distribution for X.

|  |  |  |
| --- | --- | --- |
|  | 0 | 1 |
|  | 0.9 | 0.1 |

✓

c) Determine the mode, the mean and the standard deviation of the distribution.

Mode = 0 ✓ = 0.1 ✓ 0.3 ✓

2. [ 4 marks]

The random variable has mean 40 and standard deviation 3. The random variable

Determine and if the mean and standard deviation of are 110 and 6 respectively.

So

If ✓ If ✓

✓ ✓

3. [ 5,2 = 7 marks]

is a binomial distribution with parameters and .

a) Determine and if the mean and standard deviation for are and respectively.

Formulas

So

Value of ✓

Value of ✓

Value of ✓

b) State the probability in fraction form involving indices, given the values of and in part a).

value ✓

and in fraction form ✓

4. [ 2 marks]

Suppose is a Bernoulli random variable with probability of success . Determine the value of such that is maximized. State the maximum value of .

For max

✓

✓

**GREENWOOD COLLEGE**

**Mathematics Methods Units 3 & 4**

**Test 4 Discrete Probability Distributions 2019**

Name Mark /36

**All electronic devices must be switched off and in bags.**

**Access to Formulae Sheet and one sheet of A4 notes allowed. Use of approved calculators is assumed in this section.**

**Time limit 35 minutes.**

5. [ 2,1,1,1,2 =7 marks]

It is known that 15% of Year 12 students in a large country study the equivalent of Mathematics Methods.

A random sample of n students is selected from all Year 12's in this country, and the random variable X is the number of those in the sample who study the equivalent of Mathematics Methods.

(a) Describe the distribution of X. (2 marks)

|  |
| --- |
| **Solution** |
| - binomial distribution with n trials and . |
| **Specific behaviours** |
| ✓ states binomial distribution  ✓ states parameters of binomial distribution |

(b) If , determine the probability that

(i) three of the students in the sample study the equivalent of Mathematics Methods. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ evaluates probability |

(ii) more than three of the students in the sample study the equivalent of Mathematics Methods.

(1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ evaluates probability |

(iii) none of the students in the sample study the equivalent of Mathematics Methods. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ evaluates probability |

(c) If ten random samples of 22 students are selected, determine the probability that at least one of these samples has no students who study the equivalent of Mathematics Methods. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states binomial distribution with parameters  ✓ evaluates probability |

6. [ 2,2,3,2,2,4 =15 marks]

The probability that James misses his bus to school is 0.2, and the probability that he misses the bus on any day is independent of whether he missed it on the previous day.

Over five consecutive weekdays, what is the probability that James

**a)** only misses the bus on Tuesday?

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses 0.8 for not catching bus  ✓ determines probability |

**b)** misses the bus at least twice?

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ identifies binomial situation  ✓ evaluates cumulative probability |

**c)**

misses the bus on Tuesday and on two other days?

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ identifies binomial situation for other two days  ✓ evaluates probability of missing bus on two other days  ✓ determines probability |

Christine, on the other hand, drives to work and must pass through 15 traffic lights on the way to work. She has a 0.6 probability of being stopped at any given traffic light. If she is stopped at more than 11 traffic lights she will be late for work.

**d)** Determine the probability of her being late for work on any given weekday viz. Monday to Friday.

**e)** Determine the probability of her being on time for work every week day of a given week..

✓ 0.6223132 ✓

**f)** Christine wants to increase the probability of being on time every day to at least 80%. She decides to leave home earlier and now only needs to be stopped at more than 12 traffic lights to be late for work. Has Christine achieved her goal. Justify your answer.

✓

✓ ✓

Yes she achieved 87% chance of being on time every day. ✓

7. [ 3,3 =6 marks]

When Gary’s football team adopts an attacking strategy they win 30% of the time and lose 55% of the time. When they play a defensive strategy they win 20% of the time and lose 30% of the time.

In Gary’s league a win is rewarded with 3 points, a draw 1 point and a loss no points.

**a)** Calculate the expected number of points per game for the two strategies, and determine the more successful of the two strategies on that basis..

Attacking strategy Defensive strategy

|  |  |  |  |
| --- | --- | --- | --- |
| Points | 0 | 1 | 3 |
| pos | 0.55 | 0.15 | 0.3 |

|  |  |  |  |
| --- | --- | --- | --- |
| Points | 0 | 1 | 3 |
| pos | 0.3 | 0.5 | 0.2 |

✓ ✓

Defensive is best ✓

**b)** What would be the better option if the league awarded 4 points for a win instead of 3?

Attacking strategy Defensive strategy

|  |  |  |  |
| --- | --- | --- | --- |
| Points | 0 | 1 | 4 |
| pos | 0.3 | 0.5 | 0.2 |

|  |  |  |  |
| --- | --- | --- | --- |
| Points | 0 | 1 | 4 |
| pos | 0.55 | 0.15 | 0.3 |

✓ ✓

Attacking is best ✓

8. [ 1,2,2,2,2 = 9 marks]

The probability function of a discrete uniform variable is given by

for

**a)** Show that the expected value

=

**b)** Show that the Variance

Value of ✓

formula ✓

A discrete random variable where , has a uniform distribution.

It is known that .

**c)** State the probability distribution for

Value of ✓

for Description ✓

**d)** Determine the Variance of.

= or Value of ✓

**e)** Determine

= Numerator ✓

Denominator ✓